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16 CRITERIA FOR THE DESIGN AND USE OF
AUTOMATED MISSILE GROUND EQUIPMENT
TO IMPROVE MISSILE READINESS

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CRITERIA FOR THE DESIGN AND USE OF AUTOMATED MISSILE
GROUND EQUIPMENT TO IMPROVE MISSILE READINESS.

Ever since World War II, the military services have been taking delivery on ever increasingly complex weapon systems. Part of the complexity arose because of what the weapon systems were supposed to do. For example, carrying men over inter-continental ranges at supersonic speeds is not a simple design problem. Another part arose because these systems were supposed to do things that had never been done before. I'm thinking here of the inter-continental ballistic missile. Designers were pushed to the limit of their knowledge in developing these systems. Newness. Complexity. In hardware design those two words are synonyms for trouble.

Take the inter-continental ballistic missile, the ICBM. When they attempted to launch early ICBM's they discovered some troublesome things. I'll mention but two. First, missile parts failed even when doing nothing more than standing. Now it's expensive and embarrassing to launch missiles with defective parts. One obvious solution is to check the missile before you attempt to launch it. But that leads to the second trouble. The preparatory checkout procedures required quantities of skilled manpower and took lots of time. To the Air Force, this situation was not acceptable. The Air Force doesn't have great quantities of skilled manpower and, for a quick-reacting retaliatory weapon, doesn't have lots of time.

So it was only natural for people to wonder if the notions of automation could be applied to missile checkout. They were attracted by the ideas the mathematicians had used in developing high speed computers to solve problems in arithmetic. As a result, a whole new concept was born that of automatic checkout equipment. And I'm not sure but what Pandora's box flipped its lid

over that one.

Certainly, you are all aware that even one ICBM is a complex thing. So when you study many missiles - even of one type - existing within a basing and support system, you are faced with a very large, complex problem which is beyond the grasp of the human mind. Here's where the attributes of Operations Research can be profitably used. By breaking down a large complex problem into parts or steps while still retaining a connective mechanism, by offering tools or methods for an organized, systematic attack upon these parts, an operations analyst offers aids to the solution of the overall problem.

Today, I'm going to tell you about one of these aids. I will address one part of an overall ICBM weapon system problem, that of objective criteria for the design and use of automatic checkout equipment, ^{is discussed.} Recall that the purpose of the equipment is to test the missile during its ground life - an operation which I call readiness testing. It follows that what I have to say must be imbedded in the weapon system context before overall solutions are obtained.

What is the problem for the analyst in this area of automatic checkout equipment applications? Well, let's start by making some general observations. You are familiar with the conventional numerical descriptions of military weapons. Altitudes, speeds, ranges of aircrafts; CEP's, component reliabilities and propulsive thrust of missiles are examples. Part of the problem is that often these numerical descriptions don't lend themselves readily to comparative purposes. For example, one of the most difficult problems for the military analyst is to compare aircraft and missiles. The trouble is that men in aircraft can do things that missiles can't do. And it's most difficult to establish a value for the presence of the men.

What we require from the analyst then, is a means of combining or transforming the conventional numerical descriptions into useful criteria which can be used for comparative purposes. Another part of the problem is that the conventional descriptions cut across scientific or engineering disciplines. So when a system designer has to make system decisions, he frequently falls back on intuitive reasoning, judgment, and experience. I don't mean to imply that such design practice is all bad. But I do claim that the analyst's objective aids to this decision process will not only help but, stating it more forcefully, these aids are needed. Problems of these kinds are too large and the solutions result in too much expense to rely solely on subjective guides. All right, we have made some general observations of the analyst's problem. Now let's apply them to the readiness testing problem at hand. We have noted that we must be system oriented. Therefore we know something about the type of missile and its operational and support environment. But we must also recognize that these missiles are made up of various subsystems such as guidance packages and propulsion systems. Because the subsystems are quite different in design and do noticeably different things, we should expect a requirement to tailor the design and use of automatic checkout equipment to the particular characteristics of each subsystem. The analyst's job is to develop an objective means which will tell us how to choose between various designs and uses for each subsystem. Since we check a missile to find failures that are present, we are principally concerned with failure rates, a conventional numerical description springing from reliability data. After all, reliability data are natural modes of expressing ground operating probabilities. Once we know the best test method for each subsystem, we can establish a readiness testing program for the entire missile by simply combining the individual methods.

I'll show you the criterion used in this study.

CHART 1

The statement is rather brief so suppose I talk around it for a bit. We are interested in finding failed parts in an ICBM. One might think that to have a rapid means of testing is enough. We'll test the missile often by testing the individual subsystems and repair when necessary. But this intuitive reasoning can be wrong. For example, in an extreme, one may test the missile too often and find himself wearing it out unnecessarily. In a less extreme, one may choose testing methods which actually reduce missile readiness from what it could be. Therefore, it seems more reasonable to require that the readiness testing do the best to maintain a launch ready missile over a long period of time. Stating it differently, we want maximum missile readiness at some future time. Now, as soon as we say future time, we have to back off from certainty and admit probabilities of the missile being launch ready. Note that the criterion has an operational nature; inherent are the use of the missile and its test equipment. Actually, the statement applies to testing or checkout in general. It could apply to manual means. It's only when you consider military requirements for rapid test or for remote control of the test operation that you are forced to add the word automatic to the two words checkout equipment. One final talking point, the title of this paper suggests that we want to improve missile readiness. I can rephrase that by saying that we want to maximize this criterion.

To accomplish our goal of maximum missile readiness, we must develop an objective means which will

1. Relate the goal to the factors or parameters which determine it and yet
2. Allow us to examine the individual subsystems called missile functions.

OPERATIONAL CRITERION

- THE PROBABILITY THAT THE MISSILE IS

LAUNCH READY AT SOME FUTURE TIME

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CHART 1

Only then do we have a sound basis for determining the criteria for the design and use of ACE to achieve our goal of maximum future missile readiness.

To start, we need to establish what the controlling parameters are. I don't have time to discuss them all so I'll give you a few samples.

CHART 2

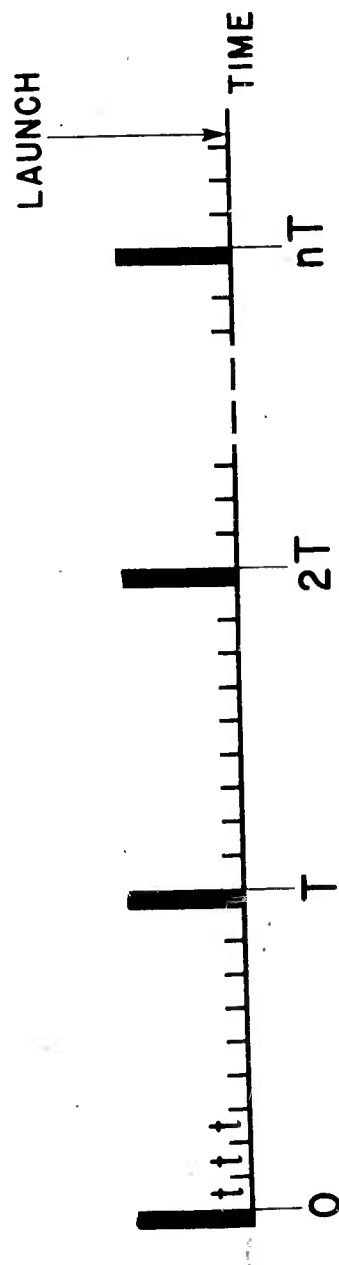
For my example, I have chosen a test environment wherein the missile is checked periodically. Certainly there are others. On the one hand I could talk about constant tests of the missile; on the other I could talk about no missile tests. We have introduced the notion of time. Therefore time is one of the controlling parameters. Among others, time for the periodic inspection, time between periodic inspections, time for maintenance delays, and for repair actions must all be included.

A second notion we have introduced is the missile being launch ready. Therefore we must consider the states of the missile. There are four. The first is launch ready or operative. The second is malfunction exists but we don't know it. The third is malfunction exists and we do know it. And the fourth is in-maintenance.

I'll show you a few other controlling parameters which are not quite so obvious.

CHART 3

CONTROLLING PARAMETERS



STATES

1. OPERATIVE
2. FAILED-UNKNOWN
3. FAILED-KNOWN
4. IN MAINTENANCE

CONTROLLING PARAMETERS

- FAILURE RATES
- FUNCTION SURVIVAL
- MALFUNCTION DETECTION
- TESTER ERRORS

The failure rates of the missile functions. The chance that the function survives the periodic inspection. The likelihood that the tester detects a missile malfunction. And finally, the chance that the tester makes a mistake by calling a good function bad. Of course, these input parameters do not affect the missile readiness probability in equal ways. Consequently the required accuracy in estimating their numerical values varies. For example, consider the first - failure rates. We'll see later on that failure rates can vary from one-tenth to one-ten thousandth. The corresponding ready probability changes from one-tenth to nine-tenths. But one would not expect an estimating error to be a factor of ten thousand. Let's say that the error in estimating a failure rate is a factor of two. We estimated a failure rate for some missile function to be 10^{-4} and later we learn that it is really 2 times 10^{-4} . Such an error would change a ready probability by less than ten per cent. On the other hand, if good tester design practice is used, then a mistake in estimating what the tester error is has a trivial effect upon the missile readiness probability. Well, there's a long list of controlling parameters. But for now, let's say that we have the list.

Our next step is to find a means of relating our goal to the long list of controlling parameters. You saw earlier that events in missile ground life are repetitive or cyclic. If we take advantage of this feature and accept exponential failure laws, then there is a branch of mathematics concerning Markov Chains which is appropriate. The mathematical model, based on compound Markov Chains, describes the movement of the missile through the four states and relates missile readiness to the list of hardware and operational parameters. In addition, the model applies whether we want to consider the missile as one piece or want to consider some individual missile

function such as a guidance package. For those of you who are mathematically inclined, I have included a few of the underlying equations at the end of the written text, P-2269.

At this point we have an operational criterion, the parameters which determine the goal, and an objective means of relating the two. Now let's see what happens when we put the three together to produce some useful results. I have two applications to show you. The first is general or generic and the second is specific.

For the generic application, we assumed characteristic ranges of numerical values for each of the parameters. Then a high speed computer determined the future probability of being ready. From the great bulk of data, I have abstracted one set to show you.

CHART 4

This set of test comparison curves is illustrative. The curves apply to one numerical set of input parameters. Change the set and you change the curves. What do we see? The curves relates ready probability to failure rates. Failure rates are given in failures per hour.

TEST COMPARISON CURVES

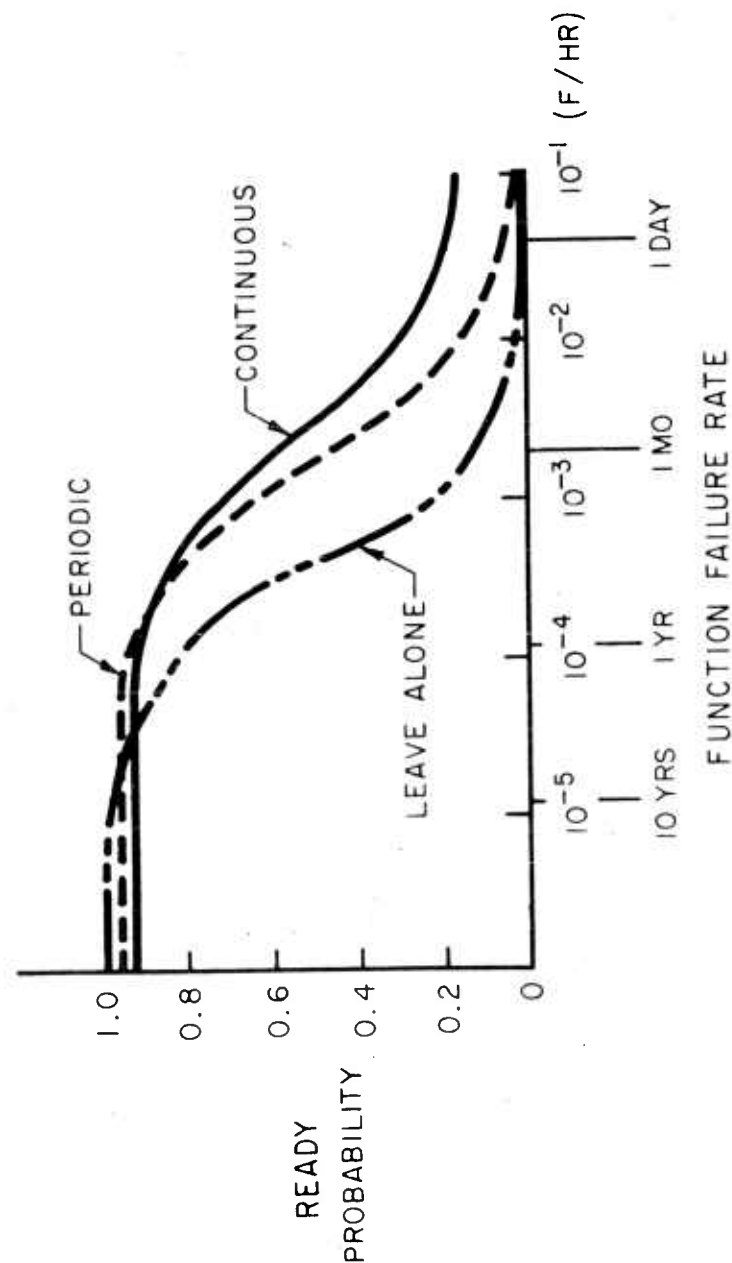


CHART 4

Underneath I have indicated a few corresponding mean-time-between-failures. This failure rate can apply to the entire missile or to some function such as the guidance package depending on the level of detail wanted. Equipment is less reliable as you go out the horizontal axis. At the origin is plotted reliable equipment and the resulting high ready probability. Out at the end are less reliable equipments with low readiness. The brown curve is a leave alone testing method where the entire missile function is replaced every six months. The green curve is a 30-day periodic check. Thirty days could be dictated by overall weapon system considerations. The orange curve is continuous monitoring of the equipment. To emphasize the illustrative intent of the chart, I'll repeat that the leave alone curve has a built in assumption of complete function replacement every six months. As the replacement period goes up in time, the ready probability goes down. Make the replacement period high enough and eventually the leave alone curve will drop beneath both the check periodic and continuous monitor curves. Not only would the chart look different but the conclusions you would draw would be different, too.

If you'll temporarily accept my set of input parameters as being realistic, we note that from the origin to slightly better than 10^{-5} leave alone is the best testing method. Best in the sense that it affords the highest ready probability. For a short while beyond, a 30-day check is best. From then on, continuous monitor is best. Now what if you don't like my set of input parameters which I have been clever enough not to show? Well, RAND

will soon have a data volume available for general use. It contains tables relating ready probabilities to the input parameters. So, select your own set, go into the data volume and plot your own test comparison curves. That alternative is really the purpose of the generic application; to give people the ability to plot their own curves as determined by their own set of input parameters.

That's all I have to say on the generic application. Before going on to the specific example, I want to use these curves to illustrate the potential harm in relying solely on intuitive choices for testing. Suppose some missile function has a 10^{-4} failure rate. This rate corresponds to an expected mean-time-between-failure of about 400 days. Well, 400 days mean-time-between-failure is over twice our replacement period of six months or 180 days. Let's leave it alone. That testing choice would result in almost a 15% reduction in readiness from what it could be with a 30-day check.

I'll go back now to my discussion of the applications. For the specific application we designed a missile; not in a disciplined engineering sense but in a component failure sense. The exemplary missile does not exist. It represents an advanced, storable propellant, inter-continental ballistic missile. The missile has two stages and a re-entry vehicle. Guidance is by an inertial system. We divided the missile into 8 major subassemblies-Chart 5 shows two.

CHART 5

FUNCTION READY - PROBABILITIES

	CONT. MON.		CHECK PER		LEAVE ALONE	
	F.R.	P.	F.R.	P.	F.R.	P.
FLIGHT CONTROL						
TIMER	.000057	.935	<10 ⁻⁶	.964	<10 ⁻⁶	1.000
PROGRAMMER	.000026	.939	<10 ⁻⁶	.964	<10 ⁻⁶	1.000
DISPLACEMENT GYROS (3)	.000032	.938	<10 ⁻⁶	.964	<10 ⁻⁶	1.000
RATE GYROS	.000110	.921	.000001	.963	.000001	.998
AMPLIFIER CHANNEL (6)	.000002	.940	<10 ⁻⁶	.964	<10 ⁻⁶	1.000
MIXER	.000174	.910	.000002	.962	.000002	.997
INERTIAL GUIDANCE PACKAGE						
STABLE PLATFORM	.000681	.788	.000681	.698	.000681	.300
COMPUTER	.001730	.638	.001730	.495	.001730	.140
TARGET TRAYS (2)	.000130	.918	.000130	.910	.000130	.755
WIRE HARNESS	.000026	.938	.000026	.958	.000026	.939
ELECTRONIC CONTROL	.001310	.718	.001310	.590	.001310	.200

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Across the top are the three testing methods: continuous monitor, check periodic, and leave alone. Beneath that are failure rates and ready probabilities. The failure rates are based on analyses of present day ICBM's. The meaning of the 10^{-6} symbol is that the failure rate is so small that it loses significance in this work. As a point of reference this number (computer .00173) corresponds to 30 days mean-time-between-failure. The ready probabilities were read from a set of working curves similar to those on the previous chart.

Consider the amplifier channel entry - .940, .964, 1.0 - the leave alone testing mode is best. The differences are due mainly to the assumed test inaccuracies and test caused failures. It's true there is a distinction in ranking but the numerical differences are small especially in light of the uncertainties and inaccuracies surrounding present-day input data. Therefore, the important point is not that leave alone is best, but rather that all three testing methods produce similar results. This similarity permits the system designer to choose between the three on different grounds than ready probabilities - cost. Certainly it must be cheaper to leave the amplifier alone than to develop, purchase, and operate automatic checkout equipment.

The situation in the inertial guidance package is quite different. By assumption, the package is running at all times to meet a military launch requirement. Operating failure rates are higher than would be the case if the unit were in the standing state of leave alone.

The stable platform ready probability for continuous monitor is over twice the value for leave alone. A factor of two is meaningful. It also points to a second consideration. Here we have units which are running. In this (L.A.) case we don't check them; here (C.M., C.P.) we do. Suppose

a group of engineers is designing a missile. They plan to turn one function on, say, to check a second function, but do not plan to check the first function itself. Such planning has occurred. This difference illustrates the penalties involved in such design practices. If you're going to run something, you better check it.

In our specific application all 8 major subsystems came out best when left alone with the exception of the inertial guidance package. There continuous monitor is a good choice. Suppose we wanted to specify a complete testing program for the entire missile. Such a program would follow the outline I just gave. Even with this best program, the ready probability for the entire missile as determined by the products of the best ready probabilities for the individual functions is a surprisingly low 0.25 caused principally by this set of ready probabilities. This low number indicates that it might be better for our exemplary missile to develop a guidance package that can be brought from rest to launch ready in a short time rather than to develop continuous monitor checking equipment. If we can do this redesign, the inertial guidance package could be kept in the standing state. The failure rates would be lower, resulting in higher ready probabilities for the inertial guidance package and a higher ready probability for the entire missile.

In summary, I have described a means of relating missile readiness to operational and hardware parameters. The results of this study are useful to an automatic checkout equipment designer in choosing the proper design and uses for such equipment. I believe the method is most useful in the preliminary design stage where design choices must be made. It can be argued that the values of the input parameters are most rough at that early stage of design. For example, failure rates based on reliability data are input

parameters. The failure rate of a computer under design is not easy to come by. But this argument doesn't negate a requirement for objective means to establish design criteria. As a design advances, input parameters improve in accuracy and the output of the model is correspondingly improved in value.

APPENDIX 1

The purpose of this section is to give the reader an introduction to the mathematics underlying this paper. The author is indebted to Mr. S. I. Firstman for permission to reproduce here part of his paper, Ref 1. A complete description of the model will be found in Ref. 2.

Problem Statement

A missile and ground-operating equipment necessary for launch are assumed to be composed of N statistically independent parts or functions. It is further assumed that the state--good or no good--of each of these functions can be determined in terms of function performance necessary to insure proper system performance. Because it forces limits for a decision of good or bad to be made in an a priori sense, this assumption neglects the chance that while a function parameter may have drifted beyond an a priori set limit, other system changes may have occurred to compensate for this drift. This undesirable feature can be alleviated by aggregating missile parts into larger test units.

It follows from the assumption of statistical independence* that the ready probability, P , for the entire weapon is given by

$$P = \prod_{i,j,k} (P_{ijk})^{x_{ijk}} \quad (1)$$

*This assumption of statistical independence does not hold strictly for all test methods. For example, using the continuous monitor method a common test item may monitor several functions. If these functions are not grouped, then the P_{ijk} estimate for each will reflect the test equipment's failure probability, and hence, P given by Eq. 1 will be a conservative estimate. When this independence assumption fails, the effects on P could be significant, but the decision process to be described will be largely unaffected. This is because each P_{ijk} is compared to each alternative P_{ijk} estimate on an

where

P_{ijk} = P (function i is operative, i.e., in ready condition
if tested using method j and equipment in location k),
and

$$x_{ijk} = \begin{cases} 1, & \text{if readiness-testing method j and equipment in} \\ & \text{location k is used for function i} \\ 0, & \text{otherwise} \end{cases}$$

To separate those functions that are constrained by safety or physical reasons from those for which design freedom still exists, let the constrained test be indicated by i_c . Then

$$P = \prod_{i=1_c} (P_{ijk})^{x_{ijk}} \prod_{i \neq 1_c} (P_{ijk})^{x_{ijk}} \quad (2)$$

$j, k \qquad j, k$

The design problem can now be stated as:

Within the constraints imposed by safety and physical reasons, and within the system readiness testing concept, determine that set of $x_{ijk} = 1$ so that P given by Eq. (2) is maximum and

$$\sum_{j,k} x_{ijk} = 1, \text{ for each } i.$$

Derivation of P_{ijk} Terms

Under the assumption that a missile function is either good or not-good at any time, the portion of the life span of a function that is spent in the silo can be divided into five states:

individual basis and in this context each estimate is, in essence, separate of the other estimates (except, of course, that all terms are based on this same system concept) because at that time, just it is being examined with all other factors being held constant.

1. Operative, there are no failures (malfunctions) in the function.
2. Inoperative--unknown; a disabling failure has occurred in the function so that it is inoperative but the failure is undiscovered.
3. Inoperative--awaiting maintenance; a disabling failure has occurred in the function, it has been discovered, and the missile is down awaiting maintenance.
4. Being maintained; the inoperative function is being repaired or replaced while the missile is down.
5. Undergoing Periodic Inspection or Preventive Maintenance; the missile is down while the function (or all functions) are being inspected or replaced.

Under these assumptions and definitions, it is necessary that all functions on a missile be in state 1, i.e., operative, in order for the missile to be ready to go.

Looking now at each function separately, it is seen that its life is a series of states with defined paths of possible transitions from state to state. If the missile is rejuvenated by preventive maintenance every several years (perhaps this period is determined by the shelf life of storable propellants or composition seals) then it appears that wear-out failures can be safely neglected in an examination of readiness testing, except that proper attention must be given to the constraints imposed on the allowable types of tests for some functions.* For example, mechanical

*Wear-out phenomena should be considered when determining the preventive maintenance concept--especially for functions that are operated continuously.

parts with definite wear-out characteristics probably should not be continuously exercised. For the readiness-test concept, the concern should be centered about the randomly occurring failures. And, for this analysis, one should also take account of the failures that are caused by the readiness testing; the different methods of testing impose different stresses on the missile function.

One further assumption is needed before the analysis can continue; that of exponentially distributed failures. The justification for this assumption and the conditions under which it is reasonable have been discussed in numerous reports (Refs. 3 for example) and will not be discussed here. This appears to be a reasonably good assumption for many physical functions, other than electronic, and there is much experimental evidence to justify its (cautious) use.

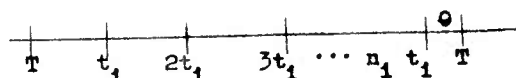
Because of the preceding assumptions on the statistics of the failure distribution, it is seen that the probability of a function failing in a given period is dependent only on the length of the period and the operational stresses, and is thereby independent of the number of periods of previous use. All other state-to-state transitions depend only on the existing state and the state to which the movement is to be made. Therefore, the progress of a particular function through its operational lifetime can now be viewed as a Markov Process which will be described.

Check Periodically

The process of checking periodically is the most tractable of the two active readiness-testing methods and will be treated as an example; the monitor continuously and leave-alone methods are discussed in Ref. 2. The length of time between periodic inspections that gives the best system

performance is assumed to be given by the system concept; this time between periodic inspections will be called T . Each missile function that uses the check periodic method is inspected every T days, and because of the nature of the system, defects that have occurred in the function can be detected only at the time of the periodic inspection.

It will prove useful to break the T days down into n_1 smaller time periods of t_1 hours each and the time required for the missile periodic inspection θ so that a periodic interval appears as



To further simplify the analysis, the quantity of time t_1 is defined to be the average length of time required to repair or replace the i th function. This includes all time required to travel to the missile, unbutton the silo, remove and repair or replace, etc.

Projecting the function's operations into the future, it appears as a series of intervals of length T , that is finally ended at the time of launch.



The movements of a missile function through this pattern must of necessity demonstrate periodic properties. The chance of a transition from one state to another will depend on whether the potential movements occur during a periodic inspection, or between inspections. It will be convenient, therefore, to define three transition matrices: A defined over t_1 , B defined over θ , and C defined over T .

For the states numbered:

1. Operative
2. Inoperative--unknown
3. Inoperative--awaiting maintenance
4. Being maintained

(NOTE: The state of being inspected is handled differently;
a transition matrix will be defined for it.)

A = matrix of single-step transition probabilities in time t_1 , for
the interval between periodic inspections.

$$A = \left\{ a_{rs} \right\} ; \text{ a matrix of terms } a_{rs}$$

where

$$a_{rs} = P (\text{transition from state } r \text{ to state } s \text{ in one interval} \\ \text{of length } t_1)$$

Similarly, for the periodic inspection

B = matrix of single-step transition probabilities in time θ , for the
periodic inspection.

$$B = \left\{ b_{rs} \right\}$$

where

$$b_{rs} = P (\text{transition from state } r \text{ to state } s \text{ during the} \\ \text{periodic inspection}).$$

The process to be described moves through n_1 steps of length t_1 described by A, and one step of length θ described by B, for each large step, or period, in its life. A matrix is needed to describe transitions over

the entire period T .

C = matrix of single-step transition probabilities in time $T = n_1 t_1 + 0$.

If

$c_{rs} = P$ (transition from state r to state s in one interval of length T),

and if it is observed that each term c_{rs} is a term compounded from a_{rs} and b_{rs} terms; that is (changing notation slightly for the sake of clarity), if $a_{rv} = p_{rv}(t_1)$, the probability of moving from state r to state v in time t_1 and $b_{vs} = p_{vs}(0)$, the probability of moving from state v to state s in time 0 , then

$$\begin{aligned} c_{rs} &= p_{rs}(n_1 t_1 + 0) \\ &= \sum_v p_{rv}(n_1 t_1) p_{vs}(0). \end{aligned}$$

By induction, it can be shown that

$$A^{n_1} = \left\{ p_{rv}(n_1 t_1) \right\},$$

and

$$B = \left\{ p_{vs}(0) \right\},$$

so

$$C = \left\{ c_{rs} \right\}$$

is given by

$$C = A^{n_1} B.$$

The next step in finding the long-run probability of being in state 1 is to define the m -step absolute probability vector $x^{(m)}$.

$$x^{(m)} = (x_1^{(m)}, x_2^{(m)}, x_3^{(m)}, x_4^{(m)})$$

where

$$x_k^{(m)} = P(\text{function being in state } k \text{ after } m \text{ periodic intervals})$$

and

$$x^{(m+1)} = x^{(m)} C$$

This vector has the property that if the process is allowed to continue sufficiently long that the effects of the initial distribution of states have been dissipated, then for some large m , a steady state, or fixed, probability vector is given by

$$x = xC \quad (14)$$

That is, the long-run distribution of states following a periodic inspection should be unaffected by an additional period and periodic inspection.

Solving for the vector x (for this problem, this involves the solution of 5 simultaneous linear equations; one for each x_k and one that expresses the sum of the x_k 's equals unity) gives the probability of the function being in each state following a periodic inspection that is conveniently long removed from the time of initial installation. In particular, this could be the inspection that precedes the beginning of hostilities.

Hostilities can begin at any randomly chosen time (with respect to the readiness testing schedule) and, therefore, a measure is needed for the

probability of each function being operative at any randomly chosen future time. If the vector describing the state probabilities in the time following a (steady state) periodic inspection is given by x , as before, then for r times periods of length t_1 later

$$x^{(r)} = xA^r \quad (15)$$

where

$$x^{(r)} = (x_1^{(r)}, x_2^{(r)}, x_3^{(r)}, x_4^{(r)})$$

Neglecting 0, for $0 < t_1$ and in turn $t_1 \ll T$, and therefore only a small error will result, the probability of the function being operative at time of launch is

$$E \left[x_1^{(r)} \right] = P_{ijk} = \frac{1}{n_1} \sum_{r=1}^{n_1} x_1^{(r)}, \text{ for each } j \text{ and } k. \quad (16)$$

The reason for this form of the expectation of $x_1^{(r)}$ over the n_1 time interval is because the launch attempt could occur at any time with equal likelihood.

The necessary matrices must now be developed using terms describing the missile function's physical characteristics, the test equipment's capabilities and error propensities, and the system operation and maintenance concepts. This is an involved and somewhat lengthy process, but not difficult. Because it is not essential to an understanding of the essence of the problem and solution, it will not be presented in this paper; it is contained in Ref. 2.

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